[논문 리뷰]

Time series analysis using variational mode decomposition with deep learning

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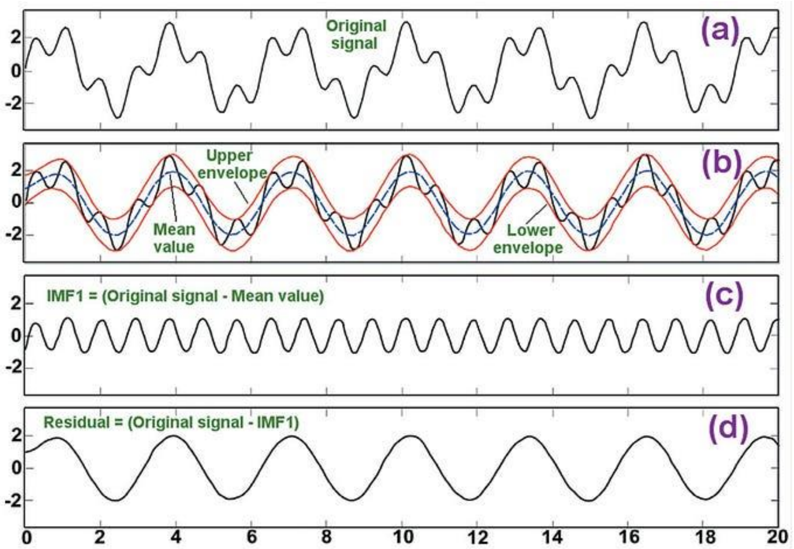
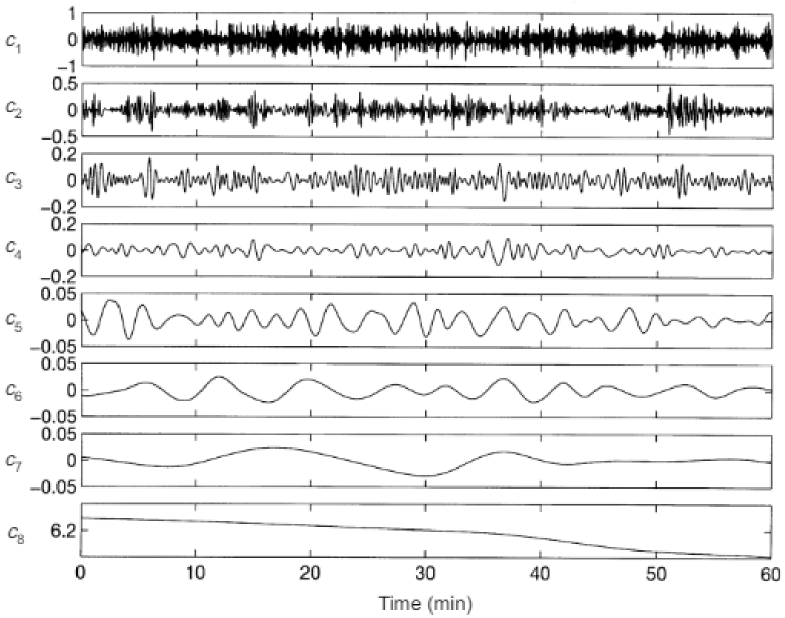
1. Empirical mode decomposition
2. Variational mode decomposition
3. Applications

Empirical mode decomposition

Empirical mode decomposition(EMD)[1]

- Fourier Transform(FT)와 달리 non-linear and non-stationary 시계열에도 작동
- Basis function($x_n(t)$)이 data-driven 형태





$$x(t) = \sum_n x_n(t) + r(t) \quad (1)$$

Sifting then means the following steps (see Fig. 1):

- Step 0: Initialize: $n := 1, r_0(t) = x(t)$
- Step 1: Extract the n-th IMF as follows:
  a) Set $h_0(t) := r_{n-1}(t)$ and $k := 1$
  b) Identify all local maxima and minima of $h_{k-1}(t)$
  c) Construct, by cubic splines interpolation, for $h_{k-1}(t)$ the envelope $U_{k-1}(t)$ defined by the maxima and the envelope $L_{k-1}(t)$ defined by the minima
  d) Determine the mean $m_{k-1}(t) = \frac{1}{2}(U_{k-1}(t) - L_{k-1}(t))$ of both envelopes of $h_{k-1}(t)$. This running mean is called the low frequency local trend. The corresponding high-frequency local detail is determined via a process called *sifting*.
  e) Form the $k-th$ component $h_k(t) := h_{k-1}(t) - m_{k-1}(t)$
     1) if $h_k(t)$ is not in accord with all IMF criteria, increase k $k+1$ and repeat the Sifting process starting at step [b]
     2) if $h_k(t)$ satisfies the IMF criteria then set $x_n(t) := h_k(t)$ and $r_n(t) := r_{n-1}(t) - x_n(t)$
- Step 2: If $r_n(t)$ represents a residuum, stop the sifting process; if not, increase n $n+1$ and start at step 1 again.

EMD Algorithm [3]

Empirical mode decomposition

Hilbert Transform

- Hilbert Transform : Real valued signal($x_i(t)$)을 complex plane($z_i(t)$)에 확장

$$\text{If } x_i(t) = \cos(w_0 t + \theta) \text{ then } z_i(t) = \cos(w_0 t + \theta) + j\sin(w_0 t + \theta) = e^{j(w_0 t + \theta)}$$

- Hilbert Transform : $H\ x_i(t) = \frac{1}{\pi} P \left\{ \int_{-} \frac{x_i(\tau)}{(t-\tau)} d\tau \right\}$

- Analytical signal $z_i(t)$: $z_i(t) = x_i(t) + iH\ x_i(t) = a_i(t)\exp(i\theta_i(t))$

$$a_i(t) = \sqrt{x_i^2(t) + H\ x_i(t)\ ^2}$$

$$\theta_i(t) = \arctan\left(\frac{H\ x_i(t)}{x_i(t)}\right)$$

$$\omega(t) = -\frac{d\theta(t)}{dt} = -\frac{d}{dt}\left(\arctan\left(\frac{H\ x(t)}{x(t)}\right)\right)$$

Empirical mode decomposition

Hilbert – Huang Transform

- IMFs can be expressed as follows : $$x_n(t) = Re\left[a_n(t)\exp\left(i\int \omega_n(t)dt\right)\right]$$
- The signal can then be expressed as follows : $$x(t) = Re\left\{\sum_{n=1}^{N} a_n(t)\exp\left(i\int \omega_n(t)dt\right)\right\} + r(t)$$
- Signal에 noise가 더해지면 복원 어려움.

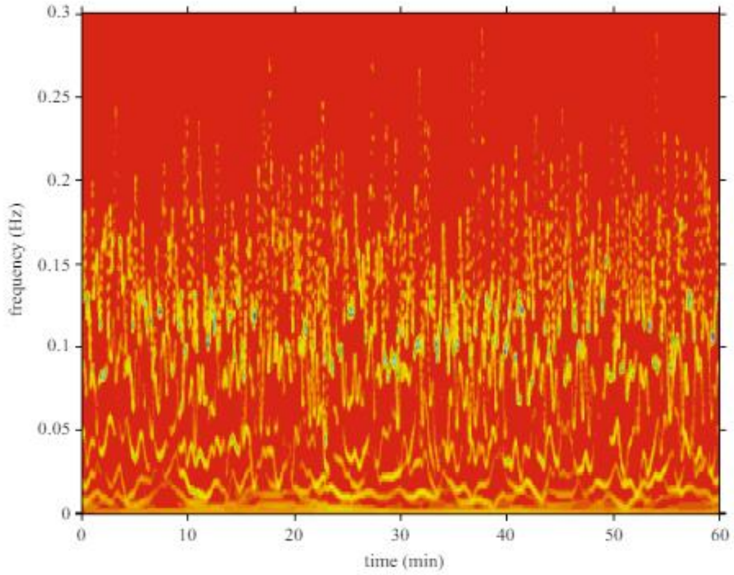


Figure 60. The 9 × 9 smoothed Hilbert spectrum for the data given in figure 58. The spectrum is extremely nodular, an indication that the wave is not stationary.

Variational mode decomposition

Variational mode decomposition (VMD) [4]

- Intrinsic Mode Function (IMF) : $x_n(t) = Re\left[a_n(t)\exp\left(i\int \omega_n(t)dt\right)\right] \gg u_k(t) = A_k(t)\cos(\phi_k(t))$
- Wiener Filtering : $f_0 = f + \eta,\ \eta \sim N(0,\sigma) \Rightarrow \min_f \left\{\|f - f_0\|_2^2 + \alpha\|\partial_t f\|_2^2\right\}$ (Least square method + Tikhonov regularization)
- Optimization problem : $\min_{\{u_k\},\{\omega_k\}} \left\{\sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t}\right) * u_k(t)\right] e^{-j\omega_k t} \right\|_2^2 \right\}$ s.t. $\sum_k u_k = f$
- The augmented Lagrangian :

$$\mathcal{L}(\{u_k\},\{\omega_k\},\lambda) := \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t}\right) * u_k(t)\right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + \left\langle \lambda(t), f(t) - \sum_k u_k(t) \right\rangle. \quad (15)$$

Application

▣ A hybrid stock price index forecasting model based on variational mode decomposition and LSTM network [5]

- HSI, SPX, FTSE, IXIC stock index에 대해 분석

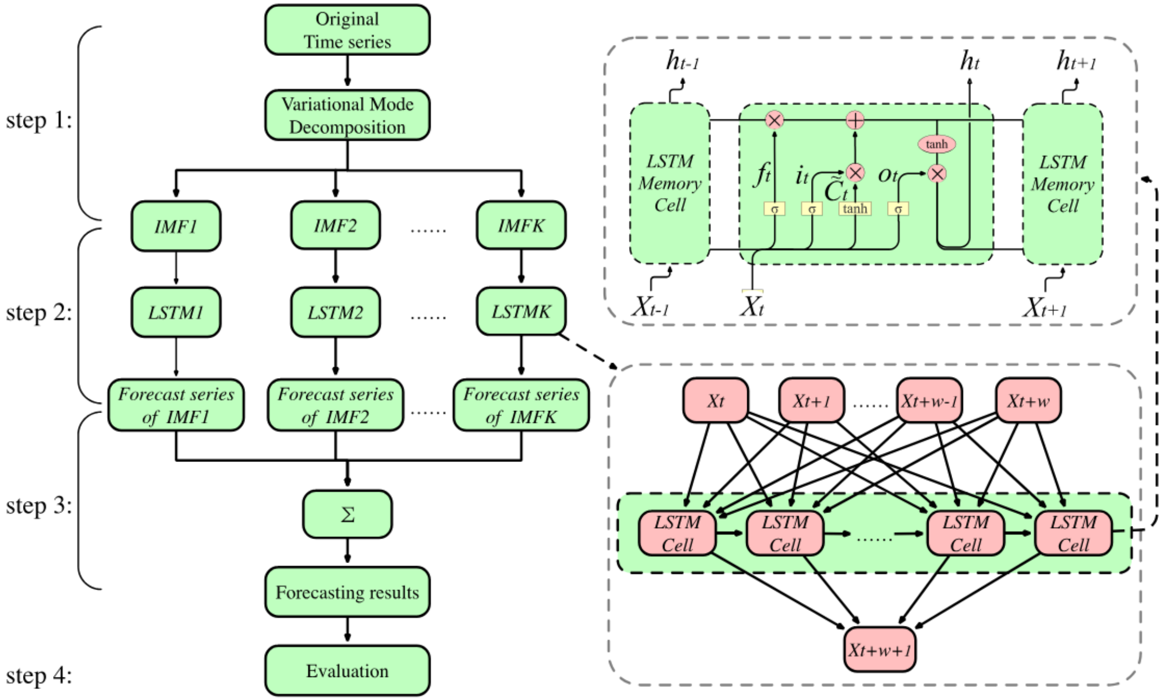
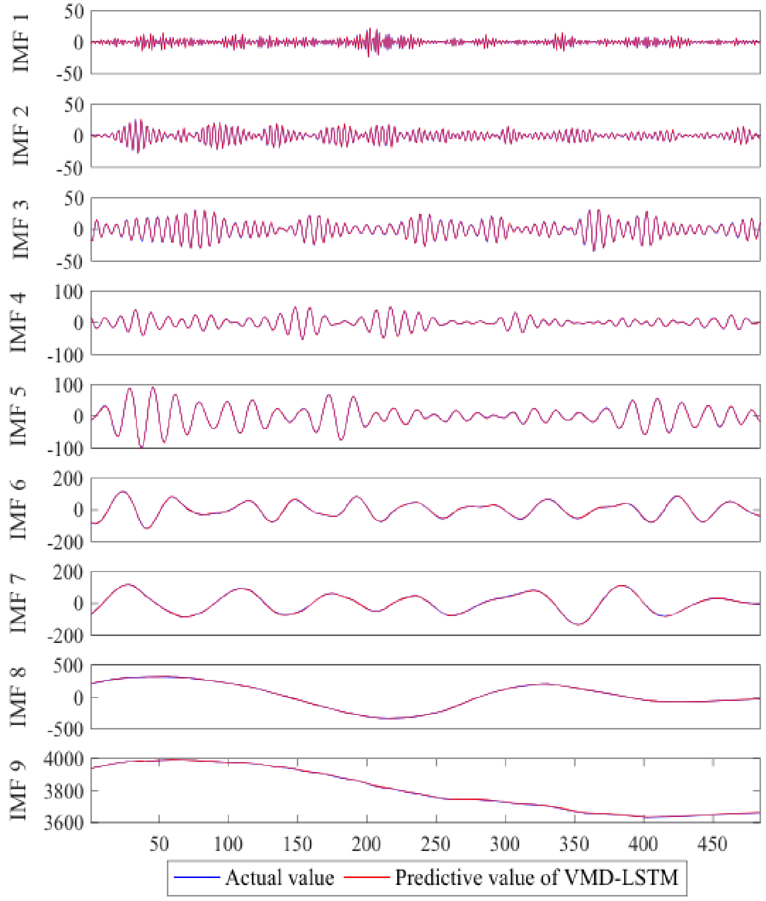


Fig. 2 Flow chart of the VMD-LSTM model



VMD-LSTM for HSI

Application

- Non-ferrous metals price forecasting based on variational mode decomposition and LSTM network [6]

  - 런던 금속거래소의 아연, 구리, 알루미늄 가격 분석

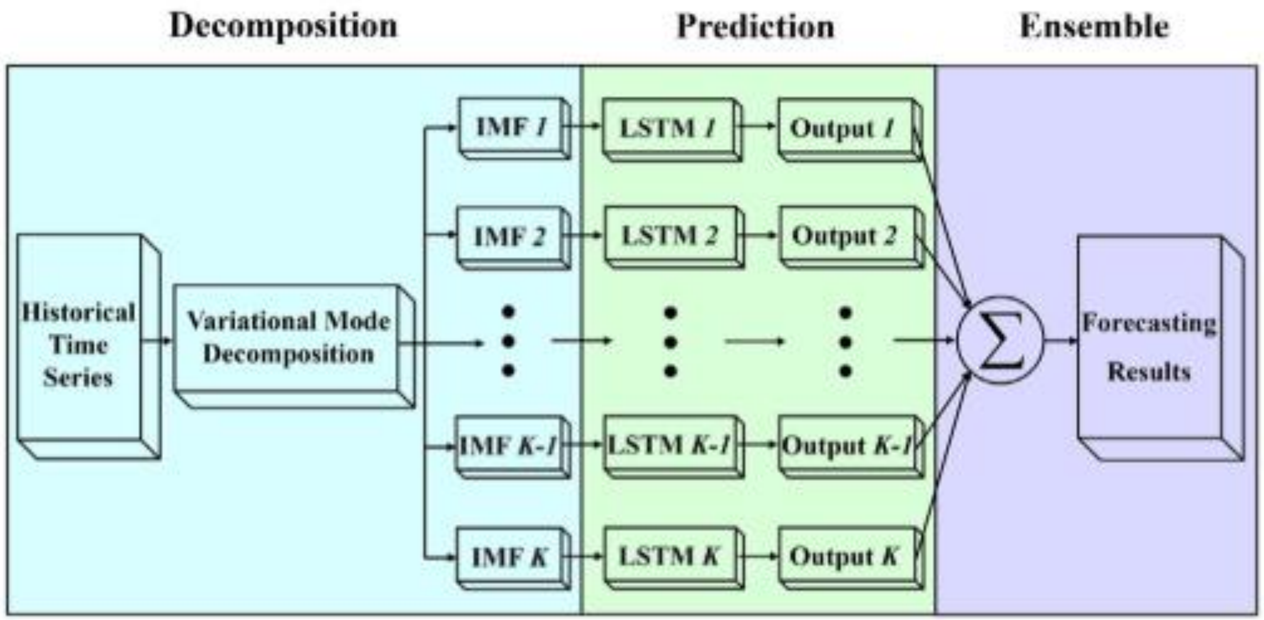


Fig. 3. The basic structure of LSTM memory block.

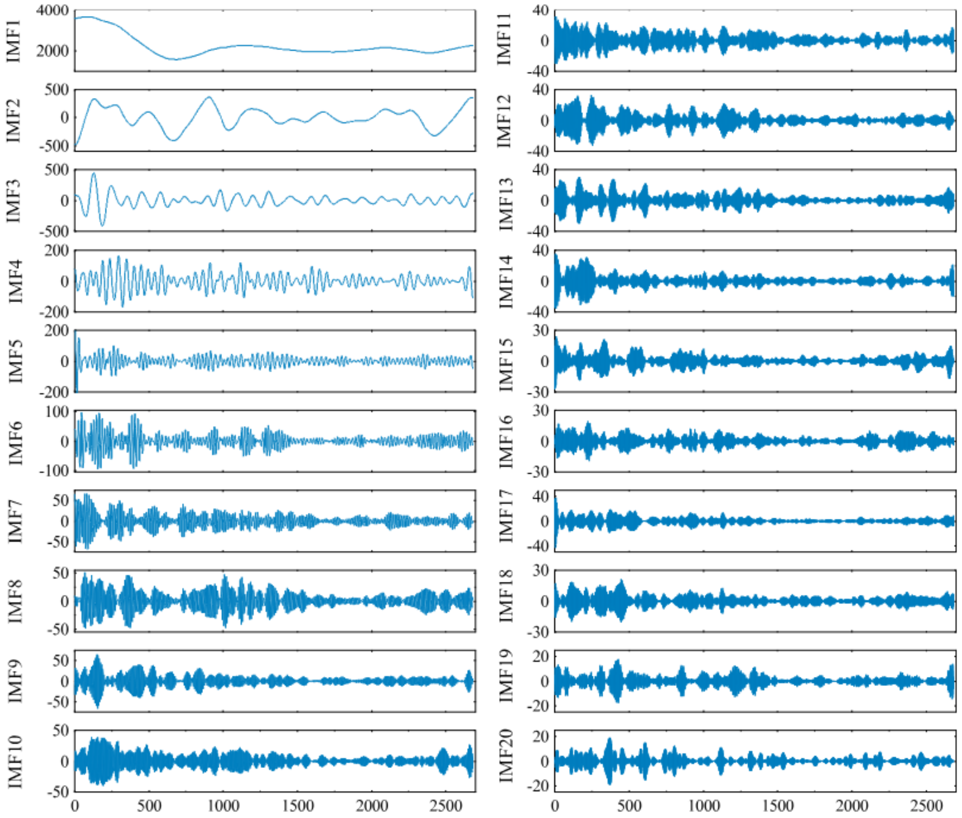


Fig. 6. The data decomposition results of LME Zinc futures price.

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