



Paper Review

N-BEATS : Neural Basis Expansion analysis for Interpretable
Time Series forecasting

Introducing NBEATSx to realized volatility forecasting

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Problem Description

- **Univariate point forecasting** problem in discrete time.
- Task is to predict the vector of length- H forecast future values by given length- T observed series history

$$[Y_1 \sim Y_T] \rightarrow [Y_{T+1} \sim Y_{T+H}]$$

Model Structure

A **deep neural architecture** based on backward and forward **residual links** and a very **deep stack of fully-connected layers**.

- **Basic building block**

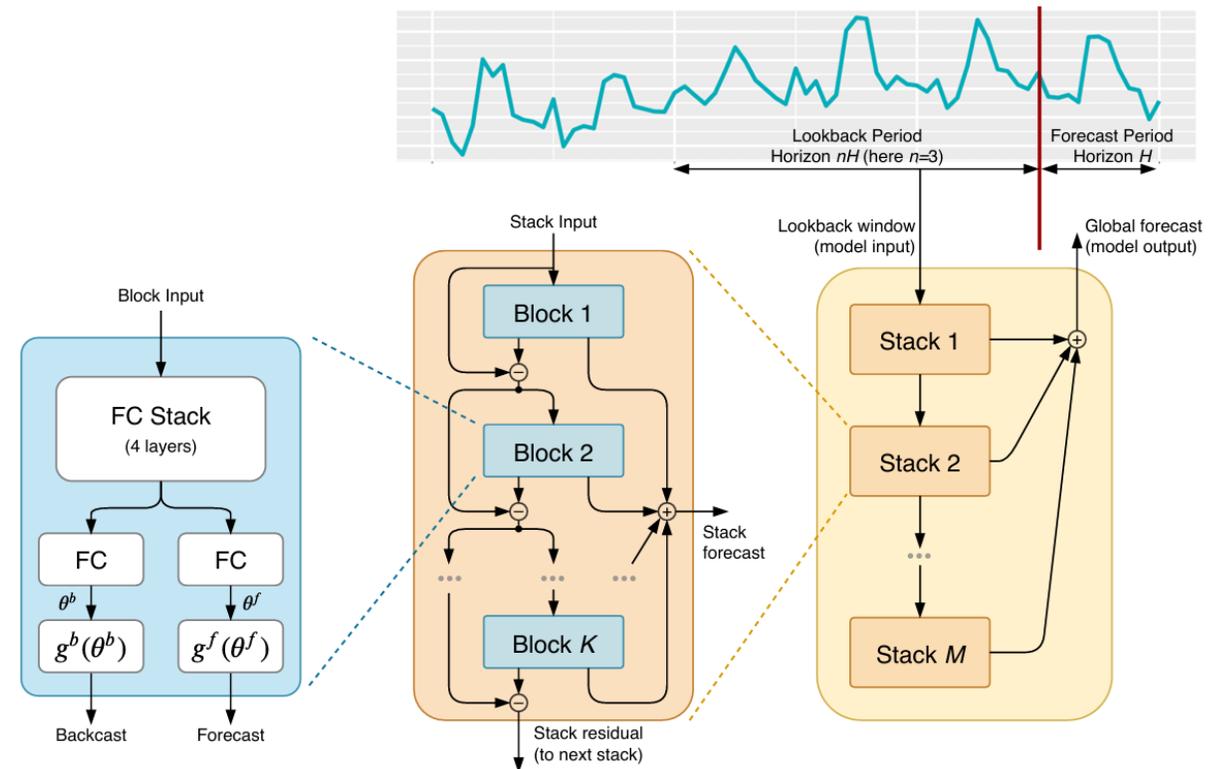
Multi-layer Full connected network with RELU nonlinearities
Predict **Basis expansion coefficients** both forward θ^f , backward θ^b

- **Doubly residual stacking**

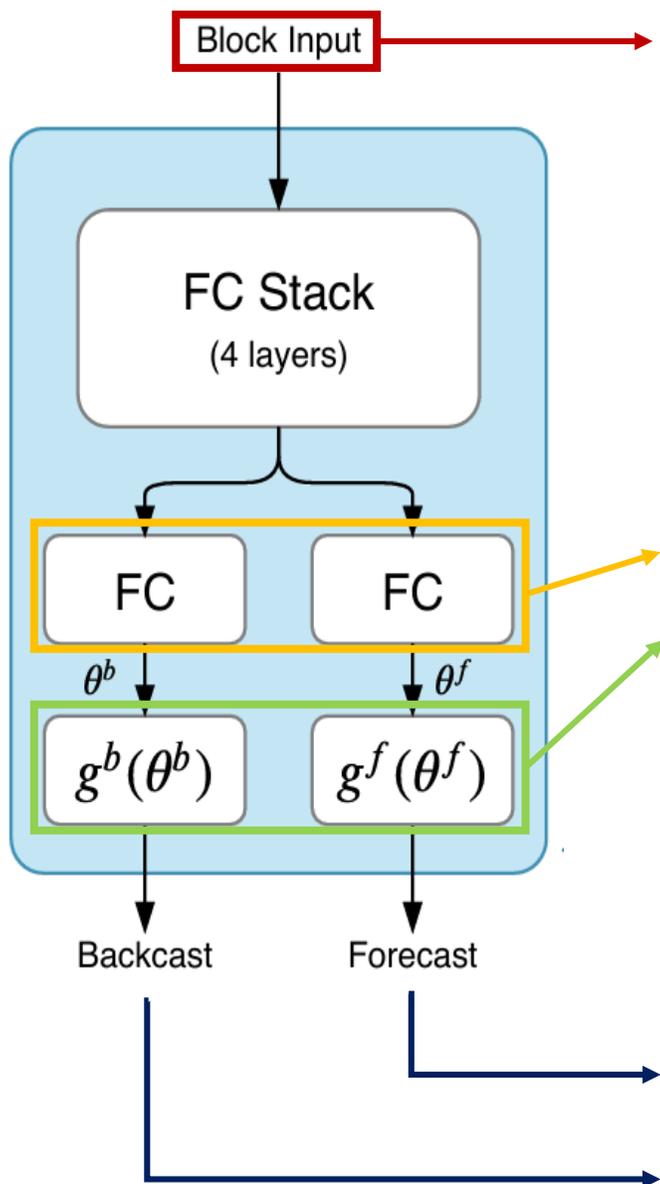
Blocks are organized into stacks using doubly residual stacking principle.
Forecasts are aggregated in hierarchical fashion.

Significance

- Empirically demonstrate that **pure DL using no time-series specific components outperforms well-established statistical approaches** on M3, M4 and TOURISM datasets
- Feasible to design an architecture with interpretable outputs that can be used as traditional decomposition techniques



Operation of Basic l-th block



Input & Output

Basic building block은 Input x_l 을 통해 output \hat{x}_l, \hat{y}_l vector 반환.

- \hat{y}_l : 목표 예측 값
- \hat{x}_l : back casting 결과

첫번째 블록의 input은 real data, 나머지 블록들의 input은 이전 블록들의 residual output

2 Parts of Block

- **First part** : forward θ^f and the backward θ^b (predictors of expansion coefficients) 를 생성하는 Fully connect network
- **Second part**: θ^f, θ^b 를 받아 basis functions g^f, g^b 와 합성하여 backcast \hat{x}_l 과 forecast \hat{y}_l 를 생성

The operation of the lth block

$$\mathbf{h}_{\ell,1} = \text{FC}_{\ell,1}(\mathbf{x}_\ell), \quad \mathbf{h}_{\ell,2} = \text{FC}_{\ell,2}(\mathbf{h}_{\ell,1}), \quad \mathbf{h}_{\ell,3} = \text{FC}_{\ell,3}(\mathbf{h}_{\ell,2}), \quad \mathbf{h}_{\ell,4} = \text{FC}_{\ell,4}(\mathbf{h}_{\ell,3}).$$

$$\theta_\ell^b = \text{LINEAR}_\ell^b(\mathbf{h}_{\ell,4}), \quad \theta_\ell^f = \text{LINEAR}_\ell^f(\mathbf{h}_{\ell,4}).$$

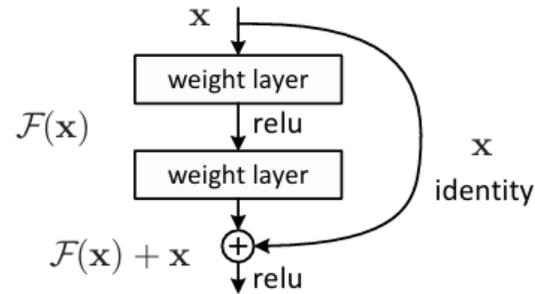
$$\hat{\mathbf{y}}_\ell = \sum_{i=1}^{\dim(\theta_\ell^f)} \theta_{\ell,i}^f \mathbf{v}_i^f, \quad \hat{\mathbf{x}}_\ell = \sum_{i=1}^{\dim(\theta_\ell^b)} \theta_{\ell,i}^b \mathbf{v}_i^b.$$

Forward expansion coefficient ultimate goal

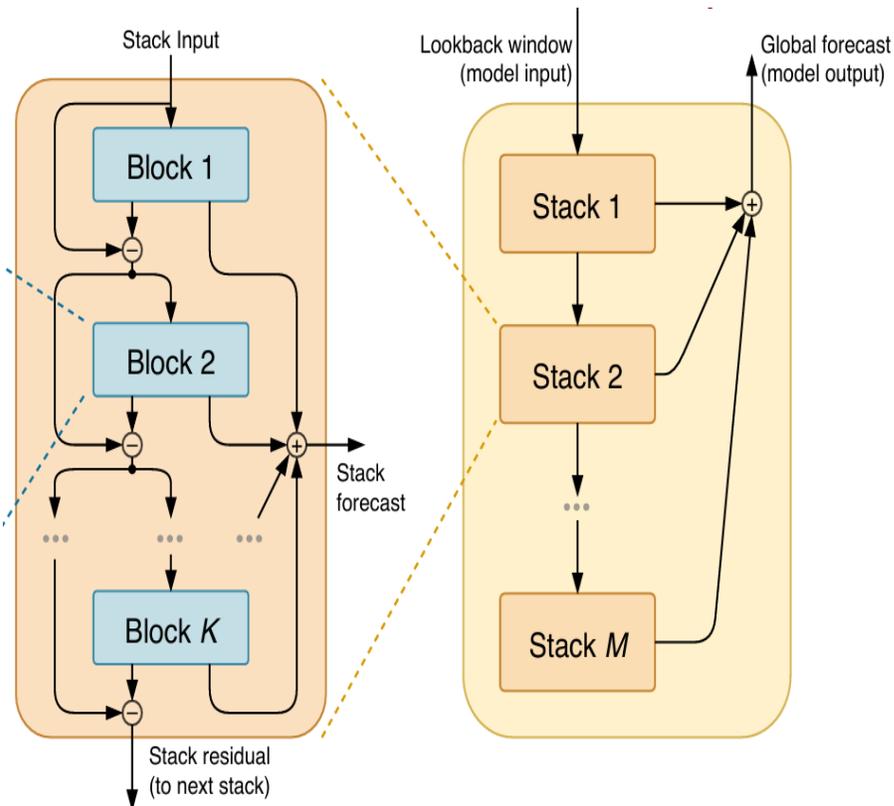
→ basis vector g_f 를 적절히 섞어 예측 Y의 정확도를 상승

Backward expansion coefficient ultimate goal

→ 예측에 도움이 되지 않는 input component를 제거해 downstream block의 forecast 도움



- The **classical residual network architecture** adds the input of the stack of layers to its output before passing the result to the next stack (He et al., 2016)
- The DenseNet architecture proposed by Huang et al. (2017) extends this principle by **introducing extra connections from the output of each stack** to the input of every other stack that follows it.
- training 성능이 향상하지만 difficult to interpret



Hierarchical doubly residual topology

두개의 residual branches를 가짐.

- Backcast prediction of each layer
- Forecast branch of each layer

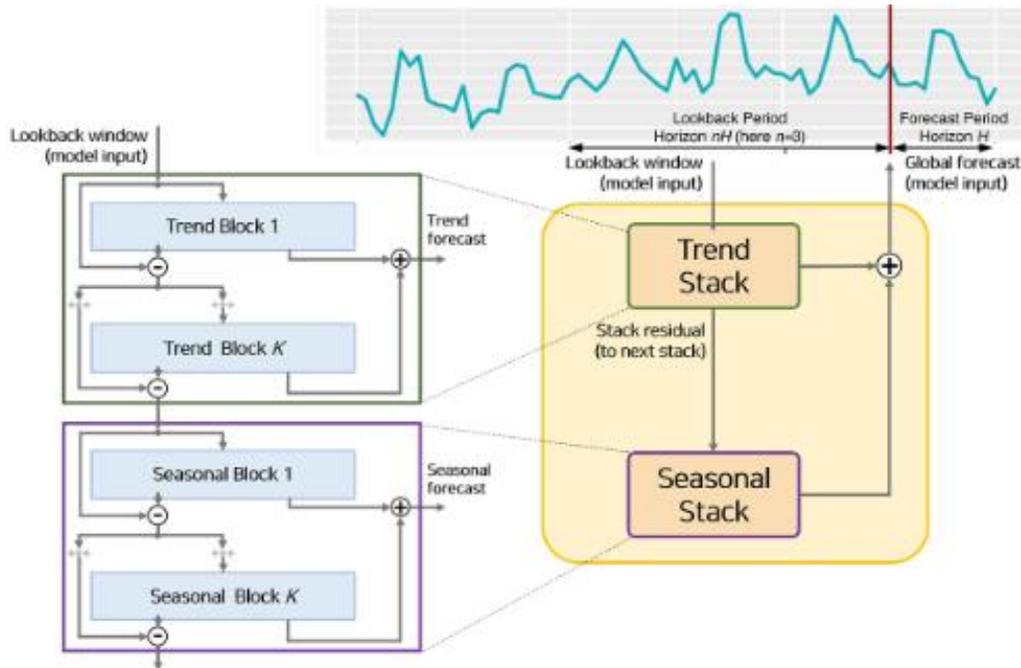
$$\mathbf{x}_l = \mathbf{x}_{l-1} - \hat{\mathbf{x}}_{l-1}, \quad \hat{\mathbf{y}} = \sum_l \hat{\mathbf{y}}_l.$$

- 다른 블록들에 대해 backcast residual branch x_l 은 순차적인 분석을 통해 계산됨
- 각각의 블록 output은 partial forecast of y 가 되어. Stack level에서 집계되고 다시 전체 네트워크 수준에서 집계됨
- The final forecast y is the sum of all partial forecasts.
- g_b, g_f (basis function)를 stack끼리 공유하게 만든 특별한 상황에서는, meaningful partial forecasts를 집계한다는 점에서 interpretability를 제공할 수 있다 주장

Generic Architecture

Basis function을 이전 layer out의 linear projection으로 설정

$$\hat{\mathbf{y}}_l = \mathbf{V}_l^f \boldsymbol{\theta}_l^f + \mathbf{b}_l^f, \quad \hat{\mathbf{x}}_l = \mathbf{V}_l^b \boldsymbol{\theta}_l^b + \mathbf{b}_l^b.$$



Inductive biases

design the **trend and seasonality decomposition** into the model to make the stack outputs more easily interpretable

푸리에 급수를 활용하여 basis function을 주기함수로 모델링

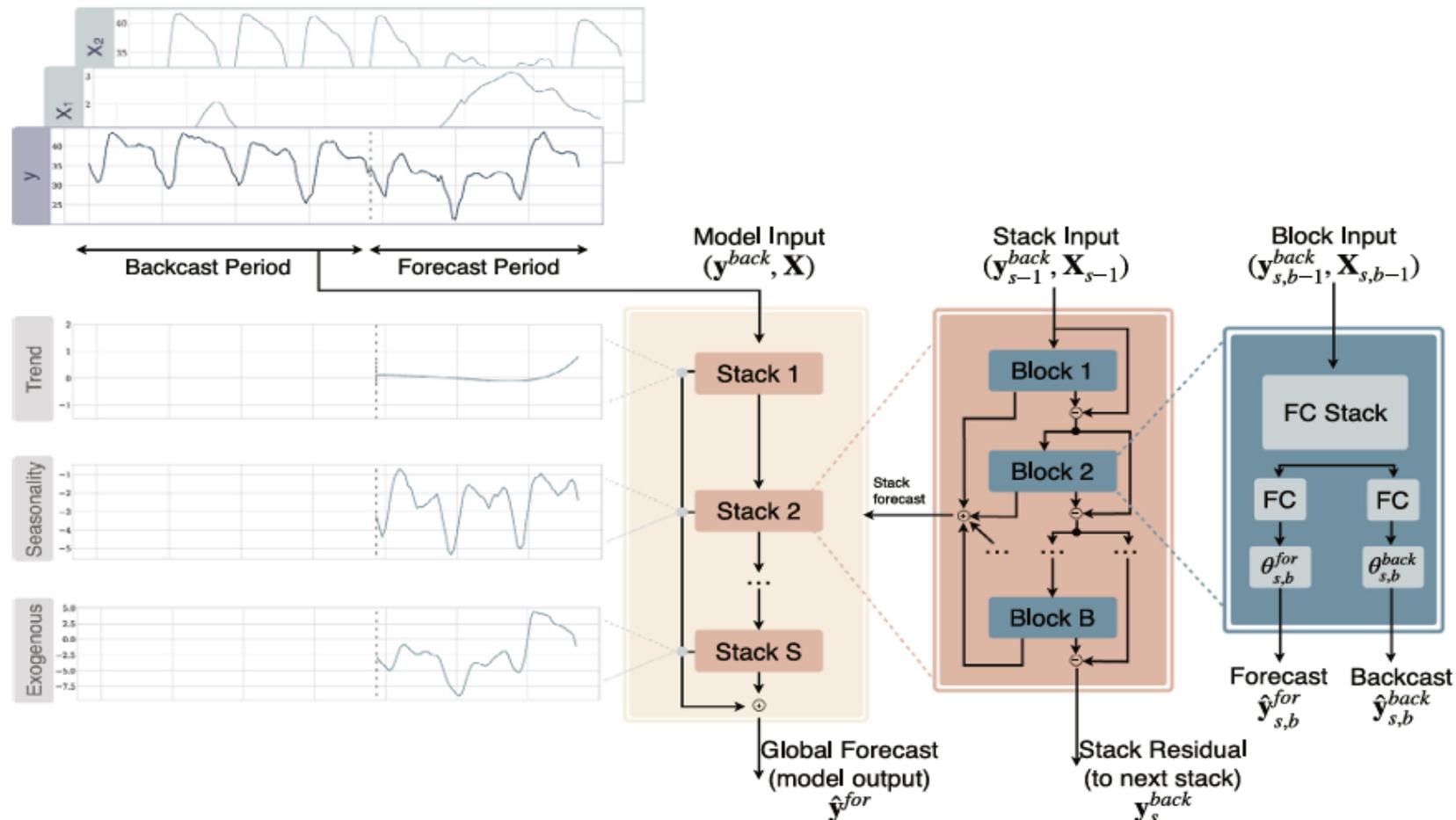
$$\hat{y}_{s,l}^t = \sum_{i=0}^{\lfloor H/2 \rfloor - 1} \theta_{s,l,i}^f \cos(2\pi t) + \theta_{s,l,i+\lfloor H/2 \rfloor}^f \sin(2\pi t),$$

전체 모델은 Trend stack과 seasonality stack을 가지게 되어 interpretable 제공

Table 1: Performance on the M4, M3, TOURISM test sets, aggregated over each dataset. Evaluation metrics are specified for each dataset; lower values are better. The number of time series in each dataset is provided in brackets.

| | M4 Average (100,000) | | M3 Average (3,003) | | TOURISM Average (1,311) | |
|-------------------|----------------------|--------------|--------------------|--------------|-------------------------|--------------|
| | SMAPE | OWA | SMAPE | | MAPE | |
| Pure ML | 12.894 | 0.915 | Comb S-H-D | 13.52 | ETS | 20.88 |
| Statistical | 11.986 | 0.861 | ForecastPro | 13.19 | Theta | 20.88 |
| ProLogistica | 11.845 | 0.841 | Theta | 13.01 | ForePro | 19.84 |
| ML/TS combination | 11.720 | 0.838 | DOTM | 12.90 | Stratometrics | 19.52 |
| DL/TS hybrid | 11.374 | 0.821 | EXP | 12.71 | LeeCBaker | 19.35 |
| N-BEATS-G | 11.168 | 0.797 | | 12.47 | | 18.47 |
| N-BEATS-I | 11.174 | 0.798 | | 12.43 | | 18.97 |
| N-BEATS-I+G | 11.135 | 0.795 | | 12.37 | | 18.52 |

- NBEATSx는 S개의 stack과 B개의 block으로 구성됨.
- NBEATS는 예측을 위해 backcast 기간에 해당하는 데이터만 input으로 사용
- NBEATSx는 외생 변수를 추가로 input으로 사용하여 longer time dependencies를 표현할 수 있도록 변환.



Purpose

N-BEATSx을 이용해 **Realized volatility(RV)**을 예측하고, 예측 성능을 다른 모델들과 비교

1. LSTM
2. TCN
3. 이질적 자기회귀(HAR 모델)
4. 일반화 자기회귀 조건부 이분산(GARCH(p,q)) 모델,
5. Glosten-Jagannathan-Runkle GARCH(GJR-GARCH(p,o,q)) 모델

Data

고려된 신경망 모델의 외생 입력 변수로 **VIX 지수**(Cboe Volatility Index) 사용

주요 데이터셋

- S&P 500, DJIA, NASDAQ 주가지수의 일별 실현 RV
- (2000년 1월 1일~ 2023년 3월 30일 | 5847개)

Robust test를 위해

- STOXX50E, IBOVESPA, S&P BSE SENSEX의 일별 RV (STOXX50E의 2007년 4월 2일부터 2023년 4월 30 |4031개)

Yang & Zhang's realized volatility

$$RV_t = \sqrt{\sigma_{o_t}^2 + k\sigma_{c_t}^2 + (1-k)\sigma_{RS_t}^2}$$

where,

$$\sigma_{o_t}^2 = \frac{1}{n-1} \sum_{t=1}^T (o_t - \bar{o})^2$$

$$\sigma_{c_t}^2 = \frac{1}{n-1} \sum_{t=1}^T (c_t - \bar{c})^2$$

RV_t =Yang & Zhang's realized volatility at time t
 o_t =opening price at time t

\bar{o} =opening price mean

c_t =close price at time t

\bar{c} = close price mean

$\sigma_{RS_t}^2$ =Rogers, Satchell, and Yoon (1994) variance estimation.

k =parameter

Yang & Zhang empirical research indicates that the best k value is given as: $k = \frac{0.34}{1.34 + \frac{n+1}{n-1}}$

- Independent of the drift(기대수익률에 독립)
- Unbiased in the continuous limit (시간간격과 관계없이 편향X)
- Consistent in dealing with opening price jumps
- 계산에 OHLC만 필요

Indicators and Tests

stochastic gradient descent optimizer algorithm의 특성으로 인해 **random seed**가 변함에 따라 **forecast accuracy** 변화

모든 변수를 고려하는 대신 15개의 표본을 추출하고 CLT를 이용해 분석을 진행

Neural network model

15 trials are performed for each model, and the mean and standard deviation of **RMSE**, **MA**, and **QLIKE** are computed to compare the forecasting performance.

Confidence Intervals (CI) of 95 % are estimated for comparison against the models

Statistical tests

두 모델 쌍의 예측 정확도가 통계적으로 유의미하게 다른지 여부를 판단하기 위해

- Diebold-Mariano(DM) 테스트
- Mann-Whitney U(MW) 테스트
- T-테스트

한 모델이 다른 모델보다 통계적으로 더 Robust forecast를 제공하는 여부를 판단하기 위해

- F-테스트

3 Robust tests

- (i) 대체 훈련 세트 크기 변경
- (ii) 신경망 모델에 상관없는 입력 변수를 추가
- (iii) 다른 지리적 지역의 주식

DM Test

두 모델의 예측 정확도가 통계적으로 유의미하게 다른지를 판단
Diebold와 Mariano(1995)에 의해 제안되었고, 이후 Harvey, Leybourne, 그리고
Newbold(1997)에 의해 시계열 의존성을 고려하도록 수정됨

- 귀무 가설 (H_0)
고려된 두 모델의 예측 사이에 통계적으로 유의미한 차이가 없다.
- 대립 가설 (H_1)
고려된 두 모델의 예측 사이에 통계적으로 유의미한 차이가 있다.

DM 테스트는 신경망 모델을 비교하는 데 일반적으로 사용되지만, 확률적 경사 하강법의 사용으로 인한 추정된 모델 매개변수와 예측 정확도의 을 고려하지 못함. 즉 확률적 경사 하강법의 무작위성에 대한 신경망 모델의 Robust 평가 불가

Mann-Whitney U (MW) 테스트, T-테스트

RMSE, MA, QLIKE 실험 결과를 사용하여 신경망 모델의 가능한 모든 쌍에 대해 테스트를 진행. 각 쌍들 사이에 예측 성능 평균에 통계적으로 유의미한 차이가 있는지를 검정

- 귀무 가설 (H_0)
특정 모델 쌍의 예측 성능 평균 사이에 통계적으로 유의미한 차이가 없다.
- 대립 가설 (H_1)
특정 모델 쌍의 예측 성능 평균 사이에 통계적으로 유의미한 차이가 있다.

F-Test

특정 모델 쌍 간 예측 정확도의 분산에 통계적으로 유의미한 차이가 있는지를 결정하기 위해 사용됩니다. 다시 말해, 이는 확률적 경사 하강법의 무작위성(즉, 다양한 무작위 시드의 선택)에 대한 신경망 모델의 Robust을 테스트

- 귀무 가설 (H_0): 모델 쌍 간 예측 정확도의 분산에 통계적으로 유의미한 차이가 없다.
- 대립 가설 (H_1): 모델 쌍 간 예측 정확도의 분산에 통계적으로 유의미한 차이가 있다.

Main sample results



Table 3
Hyperparameters Search Space.

| Hyperparameters | Neural Networks | | |
|--------------------------------------|--|-----------------------------|--|
| | NBEATSx | LSTM | TCN |
| Number of Lags | [21, 63, 84, 126, 189, 252] | [21, 63, 84, 126, 189, 252] | [21, 63, 84, 126, 189, 252] |
| Scaler Type | [robust, standard, minmax] | [robust, standard, minmax] | [robust, standard, minmax] |
| Number of Epochs | [100, 150, 200] | [1, 3, 5, 7, 10, 15] | [1, 3, 5, 7, 10, 15] |
| Number of Neurons | - | [14, 42, 56, 84, 126, 168] | - |
| Dropout Rate | [0, 0.2, 0.4] | [0, 0.2, 0.4] | [0, 0.2, 0.4] |
| Loss Functions | [MSE, MAE, MQLoss, Student T Distribution Loss] | [MSE, MAE, Huber Loss] | [MSE, MAE, Huber Loss] |
| Kernel Size | - | - | [2, 3, 4, 6] |
| Dilations | - | - | [[1, 2, 1], [1, 2, 2, 1], [1, 2, 4, 1], [1, 2, 2, 2, 1], [1, 2, 4, 2, 1], [1, 2, 4, 4, 1]] |
| Number of units of each hidden layer | [[[712, 712], [712, 712]], [[512, 512], [512, 512]], [[250, 250], [250, 250]], [[100, 100], [100, 100]]] | - | - |
| Stack Types | [[identity, identity, exogenous], [identity, trend, exogenous], [identity, seasonality, exogenous], [trend, seasonality, exogenous]] | - | - |
| Number of Harmonics | [0, 1] | - | - |
| Number of Polynomials | [0, 1] | - | - |

Table 4
Optimal Hyper parameters.

| Models | Hyperparameters | | |
|---------|---|---|---|
| | S&P 500 | NASDAQ | DJIA |
| NBEATSx | Number of lags: 21 Scaler Type: minmax Stack Types: [Identity, Identity] Number of units of each hidden layer: [[512, 512], [512, 512]] Dropout Rate: 0 Loss Function: MQLoss Number of epochs: 150 | Number of lags: 21 Scaler Type: minmax Stack Types: [Identity, Identity] Number of units of each hidden layer: [[512, 512], [512, 512]] Dropout Rate: 0 Loss Function: MQLoss Number of epochs: 200 | Number of lags: 21 Scaler Type: minmax Stack Types: [Identity, Identity] Number of units of each hidden layer: [[512, 512], [512, 512]] Dropout Rate: 0 Loss Function: MQLoss Number of epochs: 200 |
| TCN | Number of lags: 126 Scaler Type: minmax Kernel size: 4 Dilations: [1, 2, 2, 2, 1] Dropout Rate: 0.2 Loss Function: Huber Loss Number of epochs: 15 | Number of lags: 189 Scaler Type: minmax Kernel size: 3 Dilations: [1, 2, 4, 2, 1] Dropout Rate: 0.2 Loss Function: Huber Loss Number of epochs: 3 | Number of lags: 84 Scaler Type: minmax Kernel size: 6 Dilations: [1, 2, 4, 1] Dropout Rate: 0.2 Loss Function: MAE Number of epochs: 10 |
| LSTM | Number of lags: 189 Scaler Type: minmax Number of Neurons: 56 Dropout Rate: 0 Loss Function: MSE Number of epochs: 7 | Number of lags: 21 Scaler Type: minmax Number of Neurons: 168 Dropout Rate: 0 Loss Function: Huber Loss Number of epochs: 7 | Number of lags: 126 Scaler Type: minmax Number of Neurons: 126 Dropout Rate: 0 Loss Function: MSE Number of epochs: 7 |

신경망 모델과 각 주식 지수에 대해 최적의 매개변수를 결정하기 위해 사용된 하이퍼파라미터 검색 공간과 각 모델에 대해 선택된 최적

- 하이퍼파라미터 검색에 Tensorflow Keras Python 라이브러리와 T4 GPU를 사용해 **LSTM 및 TCN의 경우 평균 1시간 30분** 소요
- NBEATSx는 Nixtla Neuralforecast Python 라이브러리와 T4 GPU를 사용해 **평균 15분**

Main sample results



Table 5
Main Sample Results.

| Error Measures | | NBEATsX | TCN | LSTM | HAR | GARCH(1,1) | GJR-GARCH(1,1,1) |
|----------------|--------|-----------------------|-----------------------|-----------------------|---------|------------|------------------|
| S&P 500 | | | | | | | |
| RMSE | H = 1 | 0.246 % ≤ X ≤ 0.253 % | 0.248 % ≤ X ≤ 0.280 % | 0.235 % ≤ X ≤ 0.264 % | 0.265 % | 0.295 % | 0.295 % |
| | H = 3 | 0.245 % ≤ X ≤ 0.258 % | 0.269 % ≤ X ≤ 0.292 % | 0.257 % ≤ X ≤ 0.303 % | 0.283 % | 0.309 % | 0.307 % |
| | H = 5 | 0.261 % ≤ X ≤ 0.277 % | 0.276 % ≤ X ≤ 0.306 % | 0.275 % ≤ X ≤ 0.299 % | 0.300 % | 0.315 % | 0.316 % |
| | H = 10 | 0.284 % ≤ X ≤ 0.300 % | 0.304 % ≤ X ≤ 0.331 % | 0.298 % ≤ X ≤ 0.333 % | 0.309 % | 0.316 % | 0.314 % |
| | H = 22 | 0.332 % ≤ X ≤ 0.343 % | 0.346 % ≤ X ≤ 0.380 % | 0.327 % ≤ X ≤ 0.394 % | 0.362 % | 0.362 % | 0.353 % |
| | H = 1 | 63.78 % ≤ X ≤ 65.37 % | 52.34 % ≤ X ≤ 66.39 % | 59.67 % ≤ X ≤ 70.06 % | 60.70 % | 59.83 % | 59.63 % |
| MA | H = 3 | 62.86 % ≤ X ≤ 65.43 % | 54.90 % ≤ X ≤ 62.13 % | 48.61 % ≤ X ≤ 64.84 % | 57.50 % | 57.75 % | 57.75 % |
| | H = 5 | 60.46 % ≤ X ≤ 63.57 % | 47.94 % ≤ X ≤ 62.73 % | 49.36 % ≤ X ≤ 64.31 % | 55.27 % | 56.95 % | 56.29 % |
| | H = 10 | 58.45 % ≤ X ≤ 61.49 % | 45.69 % ≤ X ≤ 60.04 % | 45.65 % ≤ X ≤ 61.43 % | 53.28 % | 56.75 % | 55.78 % |
| | H = 22 | 50.43 % ≤ X ≤ 52.79 % | 43.76 % ≤ X ≤ 50.79 % | 38.83 % ≤ X ≤ 61.87 % | 46.09 % | 57.19 % | 56.86 % |
| | H = 1 | 8.61 % ≤ X ≤ 9.47 % | 8.38 % ≤ X ≤ 13.59 % | 7.33 % ≤ X ≤ 10.84 % | 9.19 % | 12.77 % | 12.61 % |
| | H = 3 | 8.47 % ≤ X ≤ 9.82 % | 10.50 % ≤ X ≤ 12.93 % | 9.20 % ≤ X ≤ 15.20 % | 10.51 % | 14.14 % | 13.70 % |
| QLIKE | H = 5 | 9.36 % ≤ X ≤ 10.74 % | 10.61 % ≤ X ≤ 15.66 % | 9.93 % ≤ X ≤ 15.14 % | 11.39 % | 14.22 % | 13.86 % |
| | H = 10 | 11.25 % ≤ X ≤ 12.64 % | 12.18 % ≤ X ≤ 17.33 % | 11.82 % ≤ X ≤ 17.12 % | 12.74 % | 16.17 % | 15.48 % |
| | H = 22 | 16.17 % ≤ X ≤ 17.57 % | 16.25 % ≤ X ≤ 19.07 % | 12.92 % ≤ X ≤ 20.32 % | 18.02 % | 23.99 % | 25.93 % |
| | H = 1 | 0.346 % ≤ X ≤ 0.357 % | 0.337 % ≤ X ≤ 0.396 % | 0.310 % ≤ X ≤ 0.403 % | 0.365 % | 0.422 % | 0.423 % |
| | H = 3 | 0.338 % ≤ X ≤ 0.344 % | 0.368 % ≤ X ≤ 0.388 % | 0.315 % ≤ X ≤ 0.444 % | 0.385 % | 0.449 % | 0.428 % |
| | H = 5 | 0.339 % ≤ X ≤ 0.351 % | 0.376 % ≤ X ≤ 0.415 % | 0.326 % ≤ X ≤ 0.463 % | 0.399 % | 0.437 % | 0.438 % |
| MA | H = 10 | 0.368 % ≤ X ≤ 0.383 % | 0.409 % ≤ X ≤ 0.426 % | 0.373 % ≤ X ≤ 0.488 % | 0.415 % | 0.439 % | 0.437 % |
| | H = 22 | 0.404 % ≤ X ≤ 0.417 % | 0.449 % ≤ X ≤ 0.505 % | 0.413 % ≤ X ≤ 0.546 % | 0.463 % | 0.491 % | 0.483 % |
| | H = 1 | 61.29 % ≤ X ≤ 64.07 % | 58.89 % ≤ X ≤ 67.63 % | 48.00 % ≤ X ≤ 73.65 % | 59.26 % | 56.76 % | 56.48 % |
| | H = 3 | 63.40 % ≤ X ≤ 65.33 % | 53.95 % ≤ X ≤ 65.87 % | 42.09 % ≤ X ≤ 71.17 % | 56.71 % | 54.01 % | 56.13 % |
| | H = 5 | 63.65 % ≤ X ≤ 65.59 % | 49.70 % ≤ X ≤ 63.38 % | 36.61 % ≤ X ≤ 73.05 % | 54.98 % | 56.07 % | 55.32 % |
| | H = 10 | 61.59 % ≤ X ≤ 63.48 % | 51.10 % ≤ X ≤ 57.42 % | 35.86 % ≤ X ≤ 65.35 % | 53.09 % | 56.65 % | 55.56 % |
| QLIKE | H = 22 | 57.32 % ≤ X ≤ 59.52 % | 42.25 % ≤ X ≤ 56.49 % | 34.58 % ≤ X ≤ 61.03 % | 47.93 % | 57.11 % | 57.41 % |
| | H = 1 | 9.53 % ≤ X ≤ 10.69 % | 8.75 % ≤ X ≤ 11.62 % | 6.38 % ≤ X ≤ 14.94 % | 9.81 % | 14.80 % | 14.60 % |
| | H = 3 | 8.72 % ≤ X ≤ 9.27 % | 9.57 % ≤ X ≤ 13.19 % | 6.49 % ≤ X ≤ 17.72 % | 10.70 % | 16.49 % | 14.97 % |
| | H = 5 | 8.79 % ≤ X ≤ 9.51 % | 10.58 % ≤ X ≤ 14.98 % | 5.65 % ≤ X ≤ 20.88 % | 11.26 % | 15.46 % | 14.91 % |
| | H = 10 | 10.24 % ≤ X ≤ 11.81 % | 12.94 % ≤ X ≤ 15.11 % | 9.24 % ≤ X ≤ 21.25 % | 12.45 % | 17.40 % | 16.26 % |
| | H = 22 | 14.13 % ≤ X ≤ 15.30 % | 14.65 % ≤ X ≤ 19.33 % | 12.39 % ≤ X ≤ 22.63 % | 16.36 % | 26.78 % | 27.99 % |
| DJIA | | | | | | | |
| RMSE | H = 1 | 0.242 % ≤ X ≤ 0.251 % | 0.245 % ≤ X ≤ 0.264 % | 0.227 % ≤ X ≤ 0.278 % | 0.263 % | 0.290 % | 0.291 % |
| | H = 3 | 0.235 % ≤ X ≤ 0.242 % | 0.255 % ≤ X ≤ 0.272 % | 0.246 % ≤ X ≤ 0.292 % | 0.279 % | 0.302 % | 0.305 % |
| | H = 5 | 0.242 % ≤ X ≤ 0.251 % | 0.265 % ≤ X ≤ 0.280 % | 0.256 % ≤ X ≤ 0.305 % | 0.293 % | 0.311 % | 0.312 % |
| | H = 10 | 0.264 % ≤ X ≤ 0.282 % | 0.283 % ≤ X ≤ 0.310 % | 0.287 % ≤ X ≤ 0.309 % | 0.307 % | 0.308 % | 0.307 % |
| | H = 22 | 0.315 % ≤ X ≤ 0.328 % | 0.309 % ≤ X ≤ 0.373 % | 0.315 % ≤ X ≤ 0.392 % | 0.363 % | 0.358 % | 0.351 % |
| | H = 1 | 63.51 % ≤ X ≤ 66.56 % | 61.41 % ≤ X ≤ 67.72 % | 51.68 % ≤ X ≤ 75.05 % | 61.50 % | 60.56 % | 60.29 % |
| MA | H = 3 | 65.60 % ≤ X ≤ 67.69 % | 58.45 % ≤ X ≤ 65.87 % | 51.00 % ≤ X ≤ 70.79 % | 58.58 % | 59.04 % | 58.13 % |
| | H = 5 | 64.28 % ≤ X ≤ 66.88 % | 56.88 % ≤ X ≤ 65.21 % | 47.83 % ≤ X ≤ 68.37 % | 56.73 % | 58.30 % | 57.55 % |
| | H = 10 | 61.64 % ≤ X ≤ 64.80 % | 53.25 % ≤ X ≤ 62.19 % | 52.83 % ≤ X ≤ 63.42 % | 54.58 % | 59.01 % | 58.13 % |
| | H = 22 | 51.45 % ≤ X ≤ 55.05 % | 45.77 % ≤ X ≤ 64.76 % | 35.42 % ≤ X ≤ 64.55 % | 46.86 % | 58.06 % | 57.21 % |
| | H = 1 | 8.09 % ≤ X ≤ 9.40 % | 8.41 % ≤ X ≤ 10.21 % | 5.38 % ≤ X ≤ 13.33 % | 9.00 % | 12.42 % | 12.35 % |
| | H = 3 | 7.70 % ≤ X ≤ 8.26 % | 9.17 % ≤ X ≤ 11.55 % | 7.25 % ≤ X ≤ 13.88 % | 10.20 % | 13.63 % | 13.66 % |
| QLIKE | H = 5 | 8.00 % ≤ X ≤ 8.73 % | 9.76 % ≤ X ≤ 12.28 % | 8.65 % ≤ X ≤ 15.35 % | 11.09 % | 14.45 % | 14.12 % |
| | H = 10 | 9.60 % ≤ X ≤ 10.71 % | 11.29 % ≤ X ≤ 14.21 % | 11.50 % ≤ X ≤ 13.97 % | 12.27 % | 16.06 % | 15.27 % |
| | H = 22 | 14.44 % ≤ X ≤ 15.72 % | 12.65 % ≤ X ≤ 17.73 % | 11.44 % ≤ X ≤ 21.40 % | 17.73 % | 24.98 % | 27.90 % |

The columns 'NBEATsX', 'TCN', and 'LSTM' contain 95% Confidence Intervals (CI) of the error measures results while the columns 'HAR', 'GARCH(1,1)', 'GJR-GARCH(1,1,1)' the deterministic error measures result estimated through Ordinary Least Squares (OLS) for 'HAR' and Maximum Likelihood Estimation (MLE) for 'GARCH(1,1)' and 'GJR-GARCH(1,1,1)'.

Results

- 중기 예측(H = 3 및 H = 5)
MA, RMSE 및 QLIKE에 대해 최소 7%, 8% 및 23% 더 나은 결과
- 장기 예측(즉, H = 10 및 H = 22)의 경우
MA, RMSE 및 QLIKE에 대해 최소 7.5%, 7.5% 및 10% 더 나은 결과

Robust forecast

- 단기 예측
MA, RMSE 및 QLIKE에 대해 각각 50%, 50%, 30%
- 중기 예측
MA, RMSE 및 QLIKE에 대해 각각 69%, 40%, 72%
- 장기 예측
MA, RMSE 및 QLIKE에 대해 각각 최소 84%, 43%, 51% Robust

Exceptions

전반적인 결과를 고려할 때 H = 22에 대한 GARCH 모델의 잠재적 우수성보다는 무작위성 때문일 가능성이 높음

Main sample results



Table 6
Statistic Tests Results.

| Tests | NBEATSx vs | LSTM | TCN | HAR | GARCH(1,1) | GJR GARCH(1,1,1) |
|----------|------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
| DM tests | H=1 | H ₀ | H ₁ ^{*+} | H ₁ ^{*+} | H ₁ ^{*+} | H ₁ ^{*+} |
| MW tests | | H ₀ | H ₁ ^{*+} | | | |
| T-tests | | H ₀ | H ₁ ^{*+} | | | |
| F-tests | | H ₁ ^{*+} | H ₁ ^{*-} | | | |
| | H=3 | H ₁ ^{*+} |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | H=5 | H ₁ ^{*+} |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | H=10 | H ₁ ^{*+} |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |
| | H=22 | H ₁ ^{*+} |
| | | H ₁ ^{*-} | H ₁ ^{*-} | | | |
| | | H ₁ ^{*-} | H ₁ ^{*-} | | | |
| | | H ₁ ^{*+} | H ₁ ^{*+} | | | |

*: S&P 500, -: NASDAQ, +: DJIA.

H₀ means that the null hypothesis is not rejected considering the p -value threshold of 0.01, whereas H₁ means that the null hypothesis is rejected considering the p -value threshold of 0.01.

거의 모든 주가지수와 H에 대해 대부분의 테스트에서 H₀가 기각되어 NBEATSx의 우월성을 확인

H = 1에서는 NBEATSx와 LSTM 간의 차이가 어떤 주가지수에서도 기각되지 않아 정확도 차이가 없다고 할 수 있지만, F-test를 통해 통계적으로 더 Robust하므로 여전히 NBEATSx우월

NBEATSx는 다른 모든 모델보다 통계적으로 더 정확하며 robust

Robustness test

- (i) 대체 훈련 세트 크기 변경
- (ii) 신경망 모델에 상관없는 입력 변수를 추가
- (iii) 다른 지리적 지역의 주식

Robustness test Result

- 첫 번째(i) 및 두 번째(ii) test에서 NBEATSx의 우월성은 통계적으로 유의미
- (iii)에서 NBEATSx가 STOXX50E를 사용할 때 여전히 다른 모델들을 능가
- (iii)에서 IBOVESPA와 S&P BSE SENSEX 개발도상국의 두 주가지수를 사용할 때 예측 정확도 측면에서 LSTM과 HAR 모델에 비해 우월성이 나타나지 않고, Robust 측면에서는 뛰어남.

Robustness test- Alternative training set size



Table 7
Alternative Training Set Size Results.

| Error Measures | NBEATSx | TCN | LSTM | HAR | GARCH(1,1) | GJR-GARCH(1,1,1) |
|----------------|---------|-----------------------|-----------------------|-----------------------|------------|------------------|
| S&P 500 | | | | | | |
| RMSE | H = 1 | 0.249 % ≤ X ≤ 0.262 % | 0.258 % ≤ X ≤ 0.277 % | 0.246 % ≤ X ≤ 0.286 % | 0.267 % | 0.299 % |
| | H = 3 | 0.248 % ≤ X ≤ 0.262 % | 0.244 % ≤ X ≤ 0.344 % | 0.273 % ≤ X ≤ 0.291 % | 0.284 % | 0.316 % |
| | H = 5 | 0.265 % ≤ X ≤ 0.280 % | 0.277 % ≤ X ≤ 0.315 % | 0.280 % ≤ X ≤ 0.302 % | 0.301 % | 0.322 % |
| | H = 10 | 0.292 % ≤ X ≤ 0.302 % | 0.298 % ≤ X ≤ 0.348 % | 0.296 % ≤ X ≤ 0.320 % | 0.311 % | 0.319 % |
| | H = 22 | 0.335 % ≤ X ≤ 0.343 % | 0.320 % ≤ X ≤ 0.426 % | 0.320 % ≤ X ≤ 0.352 % | 0.360 % | 0.354 % |
| MA | H = 1 | 61.31 % ≤ X ≤ 64.68 % | 57.08 % ≤ X ≤ 67.34 % | 52.96 % ≤ X ≤ 69.12 % | 60.29 % | 59.18 % |
| | H = 3 | 61.86 % ≤ X ≤ 64.40 % | 45.24 % ≤ X ≤ 70.37 % | 52.68 % ≤ X ≤ 65.61 % | 57.33 % | 56.63 % |
| | H = 5 | 60.00 % ≤ X ≤ 62.70 % | 48.22 % ≤ X ≤ 62.05 % | 50.31 % ≤ X ≤ 64.84 % | 55.25 % | 55.51 % |
| | H = 10 | 58.16 % ≤ X ≤ 60.49 % | 42.69 % ≤ X ≤ 59.76 % | 47.49 % ≤ X ≤ 63.13 % | 53.27 % | 55.10 % |
| | H = 22 | 50.70 % ≤ X ≤ 52.39 % | 33.59 % ≤ X ≤ 57.33 % | 44.89 % ≤ X ≤ 60.14 % | 46.48 % | 56.67 % |
| QLIKE | H = 1 | 9.05 % ≤ X ≤ 10.33 % | 8.80 % ≤ X ≤ 11.67 % | 7.38 % ≤ X ≤ 13.58 % | 9.23 % | 12.99 % |
| | H = 3 | 8.63 % ≤ X ≤ 10.14 % | 7.22 % ≤ X ≤ 16.98 % | 9.40 % ≤ X ≤ 13.62 % | 10.56 % | 14.71 % |
| | H = 5 | 9.43 % ≤ X ≤ 11.08 % | 10.84 % ≤ X ≤ 15.78 % | 9.94 % ≤ X ≤ 14.84 % | 11.43 % | 14.69 % |
| | H = 10 | 12.03 % ≤ X ≤ 13.25 % | 12.03 % ≤ X ≤ 18.63 % | 11.29 % ≤ X ≤ 16.33 % | 12.79 % | 16.38 % |
| | H = 22 | 16.85 % ≤ X ≤ 17.52 % | 13.58 % ≤ X ≤ 23.65 % | 12.86 % ≤ X ≤ 17.95 % | 17.92 % | 22.86 % |
| NASDAQ | | | | | | |
| RMSE | H = 1 | 0.358 % ≤ X ≤ 0.371 % | 0.349 % ≤ X ≤ 0.486 % | 0.333 % ≤ X ≤ 0.356 % | 0.364 % | 0.428 % |
| | H = 3 | 0.343 % ≤ X ≤ 0.352 % | 0.369 % ≤ X ≤ 0.484 % | 0.347 % ≤ X ≤ 0.384 % | 0.384 % | 0.460 % |
| | H = 5 | 0.347 % ≤ X ≤ 0.357 % | 0.371 % ≤ X ≤ 0.491 % | 0.362 % ≤ X ≤ 0.410 % | 0.398 % | 0.449 % |
| | H = 10 | 0.374 % ≤ X ≤ 0.385 % | 0.412 % ≤ X ≤ 0.476 % | 0.376 % ≤ X ≤ 0.449 % | 0.413 % | 0.450 % |
| | H = 22 | 0.427 % ≤ X ≤ 0.439 % | 0.450 % ≤ X ≤ 0.499 % | 0.433 % ≤ X ≤ 0.489 % | 0.459 % | 0.484 % |
| MA | H = 1 | 59.94 % ≤ X ≤ 62.38 % | 53.33 % ≤ X ≤ 67.33 % | 57.79 % ≤ X ≤ 69.34 % | 59.41 % | 55.90 % |
| | H = 3 | 62.98 % ≤ X ≤ 64.83 % | 57.16 % ≤ X ≤ 64.10 % | 52.98 % ≤ X ≤ 70.07 % | 56.94 % | 52.27 % |
| | H = 5 | 63.06 % ≤ X ≤ 64.85 % | 56.13 % ≤ X ≤ 63.40 % | 47.94 % ≤ X ≤ 67.43 % | 55.36 % | 54.01 % |
| | H = 10 | 61.77 % ≤ X ≤ 63.09 % | 52.70 % ≤ X ≤ 60.16 % | 44.29 % ≤ X ≤ 63.95 % | 53.59 % | 54.00 % |
| | H = 22 | 54.46 % ≤ X ≤ 56.56 % | 51.19 % ≤ X ≤ 57.64 % | 42.39 % ≤ X ≤ 61.08 % | 48.71 % | 54.93 % |
| QLIKE | H = 1 | 10.23 % ≤ X ≤ 11.65 % | 9.25 % ≤ X ≤ 16.73 % | 7.59 % ≤ X ≤ 11.45 % | 9.74 % | 15.18 % |
| | H = 3 | 9.18 % ≤ X ≤ 9.89 % | 10.06 % ≤ X ≤ 15.76 % | 7.88 % ≤ X ≤ 13.34 % | 10.73 % | 17.09 % |
| | H = 5 | 8.99 % ≤ X ≤ 10.01 % | 9.81 % ≤ X ≤ 16.69 % | 9.14 % ≤ X ≤ 15.43 % | 11.29 % | 15.92 % |
| | H = 10 | 10.70 % ≤ X ≤ 12.07 % | 13.27 % ≤ X ≤ 15.56 % | 10.32 % ≤ X ≤ 17.44 % | 12.49 % | 17.71 % |
| | H = 22 | 15.87 % ≤ X ≤ 18.27 % | 14.49 % ≤ X ≤ 17.82 % | 13.38 % ≤ X ≤ 18.60 % | 16.34 % | 26.33 % |
| DJIA | | | | | | |
| RMSE | H = 1 | 0.243 % ≤ X ≤ 0.250 % | 0.243 % ≤ X ≤ 0.282 % | 0.233 % ≤ X ≤ 0.293 % | 0.264 % | 0.294 % |
| | H = 3 | 0.239 % ≤ X ≤ 0.248 % | 0.255 % ≤ X ≤ 0.285 % | 0.248 % ≤ X ≤ 0.306 % | 0.280 % | 0.307 % |
| | H = 5 | 0.242 % ≤ X ≤ 0.254 % | 0.264 % ≤ X ≤ 0.289 % | 0.266 % ≤ X ≤ 0.300 % | 0.294 % | 0.317 % |
| | H = 10 | 0.269 % ≤ X ≤ 0.287 % | 0.284 % ≤ X ≤ 0.316 % | 0.265 % ≤ X ≤ 0.349 % | 0.308 % | 0.310 % |
| | H = 22 | 0.316 % ≤ X ≤ 0.334 % | 0.319 % ≤ X ≤ 0.353 % | 0.307 % ≤ X ≤ 0.364 % | 0.360 % | 0.347 % |
| MA | H = 1 | 63.10 % ≤ X ≤ 65.52 % | 61.64 % ≤ X ≤ 69.73 % | 49.70 % ≤ X ≤ 71.61 % | 61.31 % | 59.94 % |
| | H = 3 | 64.82 % ≤ X ≤ 66.76 % | 58.76 % ≤ X ≤ 67.71 % | 49.50 % ≤ X ≤ 72.93 % | 58.52 % | 57.98 % |
| | H = 5 | 63.60 % ≤ X ≤ 66.42 % | 57.33 % ≤ X ≤ 66.70 % | 52.73 % ≤ X ≤ 67.31 % | 56.75 % | 56.91 % |
| | H = 10 | 61.37 % ≤ X ≤ 64.23 % | 53.29 % ≤ X ≤ 63.22 % | 39.17 % ≤ X ≤ 72.01 % | 54.66 % | 57.61 % |
| | H = 22 | 50.14 % ≤ X ≤ 54.68 % | 50.11 % ≤ X ≤ 59.03 % | 42.39 % ≤ X ≤ 65.41 % | 47.40 % | 57.66 % |
| QLIKE | H = 1 | 8.26 % ≤ X ≤ 9.77 % | 8.28 % ≤ X ≤ 10.43 % | 6.65 % ≤ X ≤ 14.00 % | 9.00 % | 12.72 % |
| | H = 3 | 8.80 % ≤ X ≤ 8.80 % | 9.20 % ≤ X ≤ 11.32 % | 7.61 % ≤ X ≤ 14.54 % | 10.22 % | 13.93 % |
| | H = 5 | 8.08 % ≤ X ≤ 8.94 % | 9.62 % ≤ X ≤ 12.08 % | 9.45 % ≤ X ≤ 13.43 % | 11.10 % | 14.55 % |
| | H = 10 | 9.99 % ≤ X ≤ 10.91 % | 11.24 % ≤ X ≤ 14.28 % | 8.17 % ≤ X ≤ 19.61 % | 12.28 % | 15.65 % |
| | H = 22 | 14.80 % ≤ X ≤ 16.24 % | 13.53 % ≤ X ≤ 16.29 % | 11.41 % ≤ X ≤ 18.62 % | 17.58 % | 22.54 % |

The columns 'NBEATSx', 'TCN', and 'LSTM' contain 95% Confidence Intervals (CI) of the error measures results while the columns 'HAR', 'GARCH(1,1)', 'GJR-GARCH(1,1,1)' the deterministic error measures results estimated through Ordinary Least Squares (OLS) for 'HAR' and Maximum Likelihood Estimation (MLE) for 'GARCH(1,1)' and 'GJR-GARCH(1,1,1)'.

Table 8
Statistic Tests Results.

| Tests | NBEATSx vs | LSTM | TCN | HAR | GARCH(1,1) | GJR GARCH(1,1,1) |
|----------|------------|------------|------------|------------|------------|------------------|
| DM tests | H=1 | H_1^{*+} | H_1^{*-} | H_1^{*+} | H_1^{*+} | H_1^{*+} |
| MW tests | | H_1^{*-} | H_1^{*+} | | | |
| T-tests | | H_1^{*-} | H_1^{*+} | | | |
| F-tests | | H_1^{*+} | H_1^{*+} | | | |
| | H=3 | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} |
| | | H_1^{*+} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*+} | | | |
| | H=5 | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} |
| | | H_1^{*+} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*+} | | | |
| | H=10 | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} |
| | | H_1^{*+} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*+} | | | |
| | H=22 | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} | H_1^{*+} |
| | | H_1^{*-} | H_1^{*-} | | | |
| | | H_1^{*-} | H_1^{*-} | | | |
| | | H_1^{*-} | H_1^{*+} | | | |
| | | H_1^{*+} | H_1^{*-} | | | |

*: S&P 500, -: NASDAQ, +: DJIA.

H_0 means that the null hypothesis is not rejected considering the p -value threshold of 0.01, whereas H_1 means that the null hypothesis is rejected considering the p -value threshold of 0.01.

Robustness test- Uncorrelated input variable



Table 9
Uncorrelated Input Variable Results.

| Error Measures | | NBEATSx | TCN | LSTM | |
|----------------|------------|----------------------|----------------------|----------------------|----------------------|
| RMSE | S&P 500 | | | | |
| | <i>H</i> - | 0.201 % ≤ <i>X</i> ≤ | 0.217 % ≤ <i>X</i> ≤ | 0.212 % ≤ <i>X</i> ≤ | |
| | 1 | 0.207 % | 0.324 % | 0.234 % | |
| | <i>H</i> - | 0.218 % ≤ <i>X</i> ≤ | 0.226 % ≤ <i>X</i> ≤ | 0.210 % ≤ <i>X</i> ≤ | |
| | 3 | 0.230 % | 0.324 % | 0.293 % | |
| | <i>H</i> - | 0.238 % ≤ <i>X</i> ≤ | 0.257 % ≤ <i>X</i> ≤ | 0.233 % ≤ <i>X</i> ≤ | |
| | 5 | 0.244 % | 0.296 % | 0.280 % | |
| | <i>H</i> - | 0.291 % ≤ <i>X</i> ≤ | 0.281 % ≤ <i>X</i> ≤ | 0.252 % ≤ <i>X</i> ≤ | |
| | 10 | 0.300 % | 0.322 % | 0.295 % | |
| | <i>H</i> - | 0.362 % ≤ <i>X</i> ≤ | 0.309 % ≤ <i>X</i> ≤ | 0.278 % ≤ <i>X</i> ≤ | |
| | 22 | 0.372 % | 0.361 % | 0.367 % | |
| | MA | <i>H</i> - | 66.90 % ≤ <i>X</i> ≤ | 36.80 % ≤ <i>X</i> ≤ | 56.10 % ≤ <i>X</i> ≤ |
| 1 | | 69.61 % | 68.19 % | 70.96 % | |
| <i>H</i> - | | 64.09 % ≤ <i>X</i> ≤ | 36.66 % ≤ <i>X</i> ≤ | 39.20 % ≤ <i>X</i> ≤ | |
| 3 | | 67.11 % | 63.99 % | 74.75 % | |
| <i>H</i> - | | 62.27 % ≤ <i>X</i> ≤ | 39.42 % ≤ <i>X</i> ≤ | 45.97 % ≤ <i>X</i> ≤ | |
| 5 | | 64.38 % | 57.28 % | 69.58 % | |
| <i>H</i> - | | 52.61 % ≤ <i>X</i> ≤ | 34.00 % ≤ <i>X</i> ≤ | 42.23 % ≤ <i>X</i> ≤ | |
| 10 | | 54.54 % | 50.29 % | 66.61 % | |
| <i>H</i> - | | 37.64 % ≤ <i>X</i> ≤ | 22.20 % ≤ <i>X</i> ≤ | 30.37 % ≤ <i>X</i> ≤ | |
| 22 | | 39.64 % | 41.41 % | 63.63 % | |
| QLIKE | | <i>H</i> - | 7.34 % ≤ <i>X</i> ≤ | 4.36 % ≤ <i>X</i> ≤ | 6.48 % ≤ <i>X</i> ≤ |
| | | 1 | 7.80 % | 27.47 % | 13.06 % |
| | <i>H</i> - | 8.88 % ≤ <i>X</i> ≤ | 9.48 % ≤ <i>X</i> ≤ | 4.85 % ≤ <i>X</i> ≤ | |
| | 3 | 9.65 % | 22.42 % | 20.54 % | |
| | <i>H</i> - | 11.16 % ≤ <i>X</i> ≤ | 13.12 % ≤ <i>X</i> ≤ | 5.86 % ≤ <i>X</i> ≤ | |
| | 5 | 12.00 % | 20.02 % | 22.18 % | |
| | <i>H</i> - | 16.15 % ≤ <i>X</i> ≤ | 15.87 % ≤ <i>X</i> ≤ | 11.46 % ≤ <i>X</i> ≤ | |
| | 10 | 17.79 % | 23.41 % | 16.79 % | |
| | <i>H</i> - | 23.09 % ≤ <i>X</i> ≤ | 20.47 % ≤ <i>X</i> ≤ | 12.34 % ≤ <i>X</i> ≤ | |
| | 22 | 24.92 % | 29.33 % | 27.15 % | |
| | RMSE | NASDAQ | | | |
| | | <i>H</i> - | 0.280 % ≤ <i>X</i> ≤ | 0.318 % ≤ <i>X</i> ≤ | 0.255 % ≤ <i>X</i> ≤ |
| 1 | | 0.292 % | 0.532 % | 0.394 % | |
| <i>H</i> - | | 0.297 % ≤ <i>X</i> ≤ | 0.250 % ≤ <i>X</i> ≤ | 0.313 % ≤ <i>X</i> ≤ | |
| 3 | | 0.304 % | 0.592 % | 0.346 % | |
| <i>H</i> - | | 0.318 % ≤ <i>X</i> ≤ | 0.369 % ≤ <i>X</i> ≤ | 0.323 % ≤ <i>X</i> ≤ | |
| 5 | | 0.330 % | 0.428 % | 0.383 % | |
| <i>H</i> - | | 0.370 % ≤ <i>X</i> ≤ | 0.376 % ≤ <i>X</i> ≤ | 0.303 % ≤ <i>X</i> ≤ | |
| 10 | | 0.384 % | 0.468 % | 0.483 % | |
| <i>H</i> - | | 0.446 % ≤ <i>X</i> ≤ | 0.414 % ≤ <i>X</i> ≤ | 0.376 % ≤ <i>X</i> ≤ | |
| 22 | | 0.475 % | 0.511 % | 0.600 % | |
| MA | | <i>H</i> - | 65.18 % ≤ <i>X</i> ≤ | 20.40 % ≤ <i>X</i> ≤ | 34.10 % ≤ <i>X</i> ≤ |
| | 1 | 68.14 % | 68.74 % | 74.66 % | |
| | <i>H</i> - | 64.18 % ≤ <i>X</i> ≤ | 21.56 % ≤ <i>X</i> ≤ | 50.77 % ≤ <i>X</i> ≤ | |
| | 3 | 66.57 % | 67.81 % | 68.73 % | |
| | <i>H</i> - | 61.84 % ≤ <i>X</i> ≤ | 36.16 % ≤ <i>X</i> ≤ | 42.71 % ≤ <i>X</i> ≤ | |
| | 5 | 63.75 % | 55.45 % | 66.56 % | |
| | <i>H</i> - | 54.68 % ≤ <i>X</i> ≤ | 28.35 % ≤ <i>X</i> ≤ | 22.46 % ≤ <i>X</i> ≤ | |
| | 10 | 57.36 % | 54.37 % | 75.73 % | |
| | <i>H</i> - | 43.34 % ≤ <i>X</i> ≤ | 22.74 % ≤ <i>X</i> ≤ | 27.21 % ≤ <i>X</i> ≤ | |
| | 22 | 47.88 % | 43.17 % | 57.51 % | |
| | QLIKE | <i>H</i> - | 7.88 % ≤ <i>X</i> ≤ | 3.54 % ≤ <i>X</i> ≤ | 8.10 % ≤ <i>X</i> ≤ |
| | | 1 | 8.86 % | 38.20 % | 12.62 % |
| <i>H</i> - | | 8.89 % ≤ <i>X</i> ≤ | 4.05 % ≤ <i>X</i> ≤ | 8.54 % ≤ <i>X</i> ≤ | |
| 3 | | 9.74 % | 35.19 % | 16.60 % | |
| <i>H</i> - | | 11.31 % ≤ <i>X</i> ≤ | 13.92 % ≤ <i>X</i> ≤ | 9.54 % ≤ <i>X</i> ≤ | |
| 5 | | 12.54 % | 22.23 % | 16.35 % | |
| <i>H</i> - | | 14.79 % ≤ <i>X</i> ≤ | 14.86 % ≤ <i>X</i> ≤ | 11.16 % ≤ <i>X</i> ≤ | |
| 10 | | 15.86 % | 26.03 % | 18.56 % | |
| <i>H</i> - | | 22.39 % ≤ <i>X</i> ≤ | 20.12 % ≤ <i>X</i> ≤ | 15.62 % ≤ <i>X</i> ≤ | |
| 22 | | 25.54 % | 29.31 % | 25.97 % | |
| RMSE | | DJIA | | | |
| | | <i>H</i> - | 0.212 % ≤ <i>X</i> ≤ | 0.246 % ≤ <i>X</i> ≤ | 0.168 % ≤ <i>X</i> ≤ |
| | 1 | 0.221 % | 0.293 % | 0.328 % | |
| | <i>H</i> - | 0.227 % ≤ <i>X</i> ≤ | 0.260 % ≤ <i>X</i> ≤ | 0.234 % ≤ <i>X</i> ≤ | |
| | 3 | 0.233 % | 0.304 % | 0.263 % | |
| | <i>H</i> - | 0.243 % ≤ <i>X</i> ≤ | 0.260 % ≤ <i>X</i> ≤ | 0.236 % ≤ <i>X</i> ≤ | |
| | 5 | 0.248 % | 0.307 % | 0.288 % | |
| | <i>H</i> - | 0.280 % ≤ <i>X</i> ≤ | 0.274 % ≤ <i>X</i> ≤ | 0.252 % ≤ <i>X</i> ≤ | |

Table 9 (continued)

| Error Measures | | NBEATSx | TCN | LSTM |
|----------------|------------|----------------------|----------------------|----------------------|
| MA | <i>H</i> - | 0.288 % | 0.308 % | 0.322 % |
| | 10 | 0.343 % ≤ <i>X</i> ≤ | 0.294 % ≤ <i>X</i> ≤ | 0.291 % ≤ <i>X</i> ≤ |
| | 22 | 0.373 % | 0.325 % | 0.365 % |
| | <i>H</i> - | 66.21 % ≤ <i>X</i> ≤ | 42.05 % ≤ <i>X</i> ≤ | 44.93 % ≤ <i>X</i> ≤ |
| | 1 | 69.60 % | 63.55 % | 76.36 % |
| | <i>H</i> - | 65.20 % ≤ <i>X</i> ≤ | 40.66 % ≤ <i>X</i> ≤ | 49.02 % ≤ <i>X</i> ≤ |
| | 3 | 67.21 % | 58.04 % | 68.70 % |
| | <i>H</i> - | 63.65 % ≤ <i>X</i> ≤ | 41.48 % ≤ <i>X</i> ≤ | 42.95 % ≤ <i>X</i> ≤ |
| | 5 | 65.65 % | 57.79 % | 67.64 % |
| | <i>H</i> - | 57.23 % ≤ <i>X</i> ≤ | 41.31 % ≤ <i>X</i> ≤ | 42.97 % ≤ <i>X</i> ≤ |
| | 10 | 59.12 % | 56.86 % | 67.47 % |
| | <i>H</i> - | 42.35 % ≤ <i>X</i> ≤ | 38.02 % ≤ <i>X</i> ≤ | 33.52 % ≤ <i>X</i> ≤ |
| 22 | 46.90 % | 52.02 % | 62.61 % | |
| QLIKE | <i>H</i> - | 7.46 % ≤ <i>X</i> ≤ | 10.63 % ≤ <i>X</i> ≤ | 9.64 % ≤ <i>X</i> ≤ |
| | 1 | 8.51 % | 19.17 % | 32.79 % |
| | <i>H</i> - | 8.51 % ≤ <i>X</i> ≤ | 12.80 % ≤ <i>X</i> ≤ | 8.32 % ≤ <i>X</i> ≤ |
| | 3 | 9.19 % | 19.90 % | 14.49 % |
| | <i>H</i> - | 10.59 % ≤ <i>X</i> ≤ | 12.76 % ≤ <i>X</i> ≤ | 8.58 % ≤ <i>X</i> ≤ |
| | 5 | 11.51 % | 19.89 % | 18.03 % |
| | <i>H</i> - | 14.29 % ≤ <i>X</i> ≤ | 13.46 % ≤ <i>X</i> ≤ | 8.30 % ≤ <i>X</i> ≤ |
| | 10 | 15.56 % | 19.80 % | 21.14 % |
| | <i>H</i> - | 19.84 % ≤ <i>X</i> ≤ | 15.88 % ≤ <i>X</i> ≤ | 12.64 % ≤ <i>X</i> ≤ |
| | 22 | 22.57 % | 21.84 % | 23.18 % |

The columns 'NBEATSx', 'TCN', and 'LSTM' contain 95% Confidence Intervals (CI) of the error measures results while the columns 'HAR', 'GARCH(1,1)', 'GJR-GARCH(1,1,1)' the deterministic error measures results estimated through Ordinary Least Squares (OLS) for 'HAR' and Maximum Likelihood Estimation (MLE) for 'GARCH(1,1)' and 'GJR-GARCH(1,1,1)'.

Table 10
Statistic Tests Results.

| Tests | NBEATSx vs | LSTM | TCN |
|----------|------------|------------|------------|
| MW tests | H=1 | H_0^{++} | H_0^{++} |
| T-tests | | H_0^{++} | H_0^{++} |
| F-tests | | H_0^{++} | H_0^{++} |
| | H=3 | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | H=5 | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | H=10 | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | H=22 | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |
| | | H_0^{++} | H_0^{++} |

*: S&P 500, -: NASDAQ, +: DJI.

H_0 means that the null hypothesis is not rejected considering the p -value threshold of 0.01, whereas H_1 means that the null hypothesis is rejected considering the p -value threshold of 0.01.

Robustness test-Stocks from other regions



Table 11
Other Geographic Regions Results.

| Error Measures | NBEATSx | TCN | LSTM | HAR | GARCH(1,1) | GJR-GARCH(1,1,1) | |
|----------------|----------------|------------------------------|------------------------------|------------------------------|-----------------------|------------------|----------------|
| STOXX50E | | | | | | | |
| RMSE | H = 1 | 0.288 % ≤ X ≤ 0.301 % | 0.272 % ≤ X ≤ 0.346 % | 0.269 % ≤ X ≤ 0.308 % | 0.296 % | 0.332 % | 0.333 % |
| | H = 3 | 0.278 % ≤ X ≤ 0.287 % | 0.303 % ≤ X ≤ 0.356 % | 0.297 % ≤ X ≤ 0.326 % | 0.318 % | 0.355 % | 0.330 % |
| | H = 5 | 0.276 % ≤ X ≤ 0.285 % | 0.316 % ≤ X ≤ 0.378 % | 0.297 % ≤ X ≤ 0.356 % | 0.331 % | 0.361 % | 0.362 % |
| | H = 10 | 0.289 % ≤ X ≤ 0.296 % | 0.329 % ≤ X ≤ 0.404 % | 0.324 % ≤ X ≤ 0.349 % | 0.345 % | 0.362 % | 0.362 % |
| | H = 22 | 0.338 % ≤ X ≤ 0.352 % | 0.346 % ≤ X ≤ 0.490 % | 0.358 % ≤ X ≤ 0.400 % | 0.386 % | 0.404 % | 0.384 % |
| | MA | H = 1 | 61.37 % ≤ X ≤ 65.24 % | 48.62 % ≤ X ≤ 69.38 % | 55.38 % ≤ X ≤ 70.29 % | 60.52 % | 59.51 % |
| H = 3 | | 63.17 % ≤ X ≤ 66.91 % | 44.75 % ≤ X ≤ 62.99 % | 53.39 % ≤ X ≤ 64.51 % | 57.87 % | 58.41 % | 60.55 % |
| H = 5 | | 64.18 % ≤ X ≤ 67.08 % | 41.64 % ≤ X ≤ 57.39 % | 48.45 % ≤ X ≤ 64.74 % | 55.65 % | 56.96 % | 56.46 % |
| H = 10 | | 62.88 % ≤ X ≤ 66.24 % | 36.90 % ≤ X ≤ 55.02 % | 51.62 % ≤ X ≤ 60.67 % | 53.53 % | 59.25 % | 58.66 % |
| H = 22 | | 54.90 % ≤ X ≤ 58.96 % | 22.52 % ≤ X ≤ 52.19 % | 43.39 % ≤ X ≤ 55.71 % | 47.66 % | 55.45 % | 59.71 % |
| QLIKE | | H = 1 | 8.87 % ≤ X ≤ 9.97 % | 7.41 % ≤ X ≤ 14.89 % | 7.05 % ≤ X ≤ 11.85 % | 8.93 % | 12.99 % |
| | H = 3 | 8.45 % ≤ X ≤ 9.54 % | 9.76 % ≤ X ≤ 16.41 % | 9.04 % ≤ X ≤ 12.75 % | 9.66 % | 13.26 % | 13.22 % |
| | H = 5 | 8.29 % ≤ X ≤ 9.01 % | 11.78 % ≤ X ≤ 18.11 % | 9.00 % ≤ X ≤ 15.03 % | 10.63 % | 14.34 % | 14.08 % |
| | H = 10 | 8.65 % ≤ X ≤ 9.55 % | 12.95 % ≤ X ≤ 20.49 % | 11.13 % ≤ X ≤ 14.05 % | 11.40 % | 16.34 % | 15.72 % |
| | H = 22 | 12.23 % ≤ X ≤ 13.71 % | 14.42 % ≤ X ≤ 27.85 % | 13.70 % ≤ X ≤ 18.37 % | 14.29 % | 24.57 % | 21.30 % |
| | IBOVESPA | | | | | | |
| RMSE | H = 1 | 0.337 % ≤ X ≤ 0.348 % | 0.305 % ≤ X ≤ 0.412 % | 0.318 % ≤ X ≤ 0.346 % | 0.344 % | 0.400 % | 0.400 % |
| | H = 3 | 0.352 % ≤ X ≤ 0.360 % | 0.335 % ≤ X ≤ 0.399 % | 0.327 % ≤ X ≤ 0.367 % | 0.357 % | 0.394 % | 0.425 % |
| | H = 5 | 0.362 % ≤ X ≤ 0.368 % | 0.323 % ≤ X ≤ 0.457 % | 0.339 % ≤ X ≤ 0.370 % | 0.374 % | 0.408 % | 0.408 % |
| | H = 10 | 0.386 % ≤ X ≤ 0.395 % | 0.326 % ≤ X ≤ 0.495 % | 0.341 % ≤ X ≤ 0.392 % | 0.390 % | 0.417 % | 0.418 % |
| | H = 22 | 0.427 % ≤ X ≤ 0.439 % | 0.401 % ≤ X ≤ 0.451 % | 0.367 % ≤ X ≤ 0.405 % | 0.408 % | 0.394 % | 0.387 % |
| | MA | H = 1 | 68.15 % ≤ X ≤ 70.03 % | 53.85 % ≤ X ≤ 74.47 % | 63.86 % ≤ X ≤ 72.83 % | 66.06 % | 63.51 % |
| H = 3 | | 67.45 % ≤ X ≤ 68.89 % | 55.84 % ≤ X ≤ 68.81 % | 60.65 % ≤ X ≤ 71.59 % | 64.38 % | 64.04 % | 61.93 % |
| H = 5 | | 66.83 % ≤ X ≤ 68.35 % | 48.54 % ≤ X ≤ 69.34 % | 61.51 % ≤ X ≤ 71.20 % | 62.73 % | 63.03 % | 62.61 % |
| H = 10 | | 65.23 % ≤ X ≤ 66.53 % | 44.06 % ≤ X ≤ 67.67 % | 59.12 % ≤ X ≤ 72.19 % | 61.31 % | 62.93 % | 62.08 % |
| H = 22 | | 62.73 % ≤ X ≤ 64.65 % | 49.73 % ≤ X ≤ 59.11 % | 57.58 % ≤ X ≤ 68.52 % | 59.53 % | 67.95 % | 66.16 % |
| QLIKE | | H = 1 | 7.32 % ≤ X ≤ 7.79 % | 5.50 % ≤ X ≤ 12.33 % | 6.11 % ≤ X ≤ 8.63 % | 7.00 % | 10.41 % |
| | H = 3 | 7.61 % ≤ X ≤ 8.48 % | 7.38 % ≤ X ≤ 11.75 % | 6.55 % ≤ X ≤ 9.89 % | 7.52 % | 10.54 % | 10.95 % |
| | H = 5 | 8.19 % ≤ X ≤ 8.76 % | 6.97 % ≤ X ≤ 14.92 % | 6.93 % ≤ X ≤ 9.70 % | 8.03 % | 10.65 % | 10.37 % |
| | H = 10 | 9.27 % ≤ X ≤ 9.90 % | 7.62 % ≤ X ≤ 16.75 % | 6.62 % ≤ X ≤ 10.60 % | 8.61 % | 11.96 % | 11.47 % |
| | H = 22 | 12.50 % ≤ X ≤ 13.40 % | 11.05 % ≤ X ≤ 14.56 % | 7.95 % ≤ X ≤ 11.36 % | 9.56 % | 11.54 % | 11.33 % |
| | S&P BSE SENSEX | | | | | | |
| RMSE | H = 1 | 0.260 % ≤ X ≤ 0.267 % | 0.263 % ≤ X ≤ 0.297 % | 0.243 % ≤ X ≤ 0.268 % | 0.275 % | 0.316 % | 0.315 % |
| | H = 3 | 0.266 % ≤ X ≤ 0.275 % | 0.252 % ≤ X ≤ 0.363 % | 0.254 % ≤ X ≤ 0.269 % | 0.293 % | 0.319 % | 0.327 % |
| | H = 5 | 0.269 % ≤ X ≤ 0.277 % | 0.281 % ≤ X ≤ 0.361 % | 0.252 % ≤ X ≤ 0.305 % | 0.295 % | 0.333 % | 0.331 % |
| | H = 10 | 0.279 % ≤ X ≤ 0.288 % | 0.320 % ≤ X ≤ 0.378 % | 0.273 % ≤ X ≤ 0.307 % | 0.307 % | 0.323 % | 0.321 % |
| | H = 22 | 0.291 % ≤ X ≤ 0.299 % | 0.331 % ≤ X ≤ 0.415 % | 0.281 % ≤ X ≤ 0.314 % | 0.345 % | 0.321 % | 0.341 % |
| | MA | H = 1 | 68.96 % ≤ X ≤ 70.97 % | 61.17 % ≤ X ≤ 70.35 % | 64.90 % ≤ X ≤ 74.23 % | 65.72 % | 64.57 % |
| H = 3 | | 68.66 % ≤ X ≤ 70.40 % | 47.06 % ≤ X ≤ 70.25 % | 66.65 % ≤ X ≤ 73.27 % | 63.05 % | 63.06 % | 62.35 % |
| H = 5 | | 68.57 % ≤ X ≤ 70.19 % | 46.48 % ≤ X ≤ 65.36 % | 57.77 % ≤ X ≤ 72.78 % | 62.73 % | 61.58 % | 61.35 % |
| H = 10 | | 67.61 % ≤ X ≤ 69.58 % | 43.59 % ≤ X ≤ 56.53 % | 59.45 % ≤ X ≤ 70.44 % | 61.18 % | 64.16 % | 63.61 % |
| H = 22 | | 67.33 % ≤ X ≤ 68.80 % | 37.29 % ≤ X ≤ 54.69 % | 59.43 % ≤ X ≤ 70.18 % | 57.80 % | 66.91 % | 64.25 % |
| QLIKE | | H = 1 | 7.99 % ≤ X ≤ 9.24 % | 7.24 % ≤ X ≤ 9.75 % | 5.75 % ≤ X ≤ 8.16 % | 7.69 % | 10.57 % |
| | H = 3 | 8.59 % ≤ X ≤ 9.28 % | 6.83 % ≤ X ≤ 14.80 % | 6.44 % ≤ X ≤ 7.82 % | 8.36 % | 10.49 % | 11.45 % |
| | H = 5 | 9.34 % ≤ X ≤ 9.78 % | 8.77 % ≤ X ≤ 15.26 % | 6.28 % ≤ X ≤ 10.78 % | 8.75 % | 12.35 % | 11.88 % |
| | H = 10 | 10.06 % ≤ X ≤ 10.78 % | 11.82 % ≤ X ≤ 16.91 % | 7.32 % ≤ X ≤ 10.38 % | 9.58 % | 13.40 % | 12.76 % |
| | H = 22 | 13.01 % ≤ X ≤ 14.81 % | 12.43 % ≤ X ≤ 19.35 % | 7.89 % ≤ X ≤ 10.44 % | 10.84 % | 15.80 % | 14.33 % |

The columns 'NBEATSx', 'TCN', and 'LSTM' contain 95% Confidence Intervals (CI) of the error measures results while the columns 'HAR', 'GARCH(1,1)', 'GJR-GARCH(1,1,1)' the deterministic error measures results estimated through Ordinary Least Squares (OLS) for 'HAR' and Maximum Likelihood Estimation (MLE) for 'GARCH(1,1)' and 'GJR-GARCH(1,1,1)'.

Table 12
Statistic Tests Results.

| Tests | NBEATSx vs | LSTM | TCN | HAR | GARCH(1,1) | GJR GARCH(1,1,1) |
|----------|------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
| DM tests | H=1 | H ₀ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ |
| MW tests | | H ₀ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ |
| T-tests | | H ₀ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ |
| F-tests | | H ₀ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ | H ₁ ⁺⁺ |
| | H=3 | H ₁ ⁺⁺ |
| | H=5 | H ₁ ⁺⁺ |
| | H=10 | H ₁ ⁺⁺ |
| | H=22 | H ₁ ⁺⁺ |

H₀ means that the null hypothesis is not rejected considering the *p*-value threshold of 0.01, whereas H₁ means that the null hypothesis is rejected considering the *p*-value threshold of 0.01.

*: STOXX50E, -: IBOVESPA, +: S&P BSE SENSEX.

NBEATSx를 활용한 bitcoin price or 통계지표 prediction

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